University of Bahrain

College of Information Technology
Department of Computer Science

ITCS253 Discrete Structures II

Second Semester 2015/2016

Second Exam – 70 Minutes

SERIAL

***** Key Solution *****

STUDENT NAME	****** Key Solution ******
STUDENT#	****** Key Solution ******
SECTION	****** Key Solution ******

- This exam contains **5 pages** (including this cover page) and **4 questions**. Check to see if any pages are missing.
- You are **allowed** to use Calculators.
- You are **not allowed** to use books, notes, or mobiles.
- Please write **one answer**. In case of writing multiple answers, mistakes in any answer will be counted.

Question	Points	Score
1	7	
2	6	
3	9	
4	8	
Total:	30	

Instructor: Dr. Ali Alsaffar Sections# 1 & 2

Answer all questions

(1) Answer the following questions.

(a) **[1 point]** Give an example of an *inhomogeneous* recurrence relation a_n if its characteristic equation has the roots -1 and 4.

Solution: Since the roots are -1 and 4, then the characteristic equation is $(x+1)(x-4)=0$. Hence, $a_n + a_{n-1} = 4^n$ or $a_n - 4a_{n-1} = (-1)^n$.

(b) **[2 points]** Is the below recurrence relation linear and/or inhomogeneous? Why?

$$\frac{4}{a_{n-1}} + 3 = \frac{a_n}{a_{n-1}}$$

Solution: It's inhomogeneous and linear. How?
Multiply both sides by $a_{n-1} \implies 4 + 3a_{n-1} = a_n \implies \therefore a_n - 3a_{n-1} = 4$.

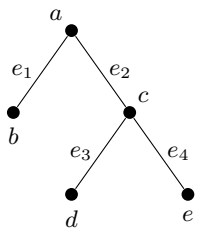
(c) **[1 point]** Why can not exist a graph whose degree sequence is 5,4,4,3,2,1?

Solution: Because sum of the degrees $5+4+4+3+2+1 = 19$ is not even.

(d) **[1 point]** Why a subgraph of a tree is not necessarily a tree? Give a reason.

Solution: Because it can be disconnected.

(e) **[2 points]** Write the incident matrix of the tree below.



Solution:

$$A_T = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

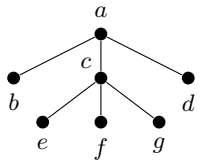
(2) Let T be a full *m-arry* tree with e edges.

(a) **[4 points]** Show that $i = e/m$ and $t = \dfrac{e(m-1)}{m} + 1$,
where i is the number of internal vertices and t is number of terminals (leaves.)

(b) **[2 points]** Draw a full $3 - arry$ tree that has 6 edges. **Hint**: Use part (a) to get i and t.

Solution:

1. For any tree with n vertices, we know that $n = e + 1$. Also, for a full *m-arry* tree, we have $n = mi + 1$. Then, $e + 1 = mi + 1 \implies e = mi \implies \therefore i = e/m$.
   Since $mi + 1 = i + t$, then $t = (m-1)i + 1$.
   Substitute for $i = e/m$, we get $\implies t = \dfrac{e(m-1)}{m} + 1$.
2. $m = 3, e = 6$. Then, $i = e/m = 6/3 = 2$, $n = e + 1 = 6 + 1 = 7$ and
   $t = \dfrac{6(3-1)}{3} + 1 = 5$.



(3) (a) **[6 points]** Use the characteristic equation to solve the following recurrence relation.

$$t_0 = 1, t_1 = -4 \quad t_n = -6t_{n-1} - 9t_{n-2}, \quad n \geq 2$$

Solution:

By re-writing the relation: $t_n + 6t_{n-1} + 9t_{n-2} = 0$

The characteristic equation is $x^2 + 6x + 9 = (x+3)^2 = 0$

$\therefore t_n = c_1(-3)^n + c_2 n(-3)^n$

From boundary conditions:

$t_0 = 1 \rightarrow \quad c_1 = 1$

$t_1 = -4 \rightarrow \quad -3c_1 - 3c_2 = -4$

and $c_2 = (-4+3)/3 = -1/3$.

Hence, $t_n = (-3)^n - n(-3)^{n-1}$.

(b) **[3 points]** Use the characteristic equation to solve the following recurrence relation without finding the constants.

$$a_n = 5a_{n-1} + (-1)^n(n+1)^3$$

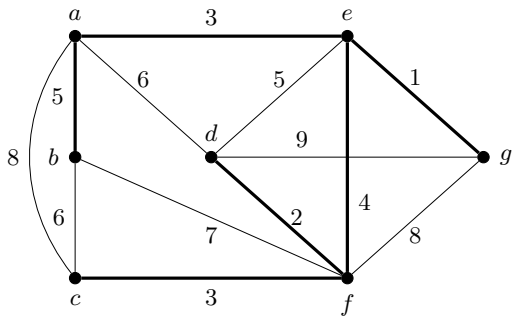
Solution:

$a_n - 5a_{n-1} = (-1)^n(n+1)^3$.

The characteristic equation is $(x-5)(x+1)^4 = 0$.

Then, $a_n = c_1(5)^n + c_2(-1)^n + c_3 n(-1)^n + c_4 n^2(-1)^n + c_5 n^3(-1)^n$.

(4) Consider the following graph.



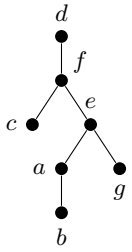
Find the minimum spanning tree for the graph starting from the vertex d.

(a) **[4 points]** In the table below indicate the order in which edges are added to form the spanning tree.

Edge	df	fc	fe	eg	ea	ab			
Weight	2	3	4	1	3	5			

(b) **[1 point]** Draw the spanning tree as a rooted tree with vertex d as the root.

Solution:



(c) According to the rooted tree found in part (b), answer the following questions.

(1) **[1 point]** As an m-arry tree, what is the value of m?

Solution: $m = 2$.

(2) **[1 point]** Is the tree a full m-arry tree? Justify your answer.

Solution: No. Not every internal vertex has exactly 2 children.

(3) **[1 point]** List all the internal vertices of the tree.

Solution: d, f, e, and a.